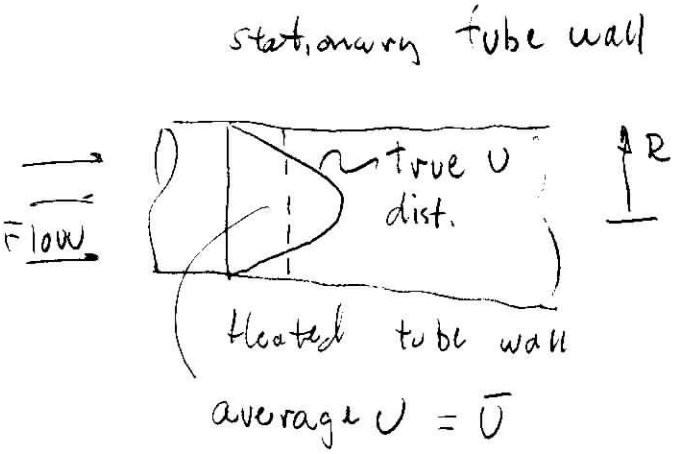
Internal Flows

Ducts ○ □ △

℄ usually



In fully developed region (far down pipe)

Recall $\dot{m} = \int_{A_c} \rho U(r)\, dA_c = \rho \bar{U} A_c$

or

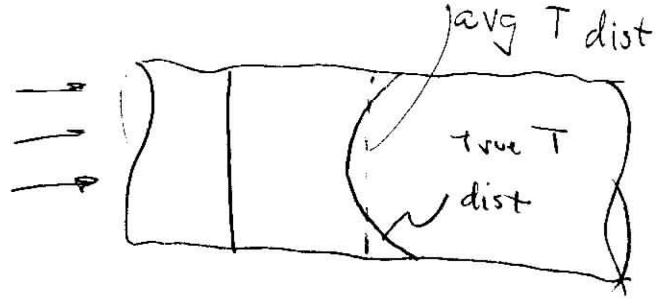
$$\bar{U} = \frac{\int_{A_c} \rho U(r)\, dA_c}{\rho A_c} = \frac{\int_{r=0}^{R} \rho U(r)\, 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2}\int_{r=0}^{R} U(r)\, r\, dr$$

gotta know

For energy transport down pipe

$$\dot{E}_{fluid} = \dot{m} C_p \bar{T}$$

$$= \int_{A_c} \rho C_p T(r)\, U(r)\, dA_c$$



so

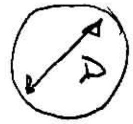
$$\bar{T} = \frac{\int_{r=0}^{R} \rho C_p T(r)\, U(r)\, 2\pi r\, dr}{\underbrace{\rho \bar{U} (\pi R^2) C_p}_{\dot{m} C_p}} = \frac{2}{\bar{U} R^2}\int_{r=0}^{R} T(r)\, U(r)\, dr$$

Usually evaluate mtl. properties @ a bulk temperature ($\approx \bar{T}$)

$$T_{bulk} = \frac{T_{inlet} + T_{exit}}{2} = \frac{T_i + T_e}{2}$$

Hydraulic Diameter

In a circular tube $\quad Re = \dfrac{\bar{U} D}{\nu} = \dfrac{\rho \bar{U} D}{\mu} = \dfrac{4\dot{m}}{\mu \pi D}$



Recall $\nu = \mu/\rho$

We call the Hydraulic dia $\quad D_{hyd} \equiv \dfrac{4 A_{cross}}{perim.}$

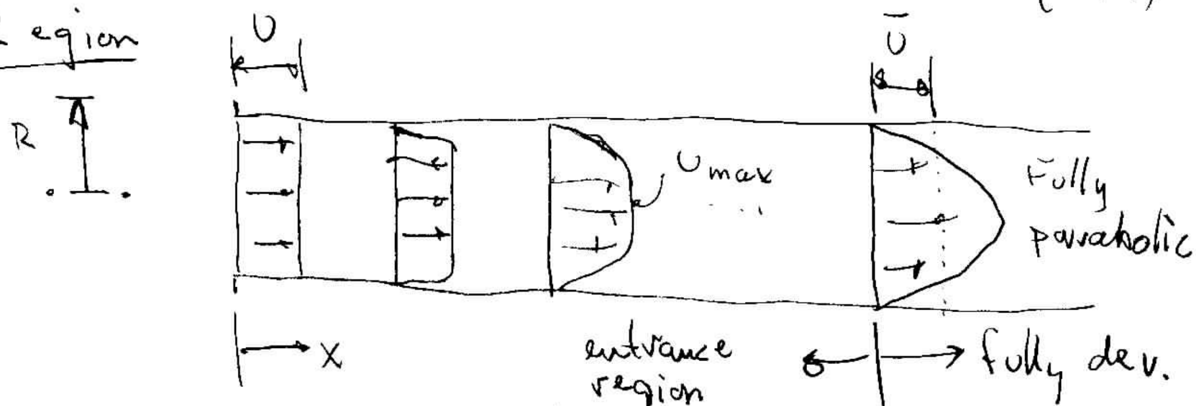
so for a cyl. $\quad D_h = \dfrac{4\pi\left(\frac{D^2}{4}\right)}{\pi D} = D$

Usually for a circ. tube

$Re < 2000$	$Re \sim 4000$ (most likely Turb.)	$Re > 10{,}000$
prob. lam. $\leftarrow$	Transition lam. to Turb.	$\rightarrow$ fully turb.

Special cases ... lam. flow has been maintained upto $Re \sim 100{,}000$ (\$\$!)

Entrance Region



Note $\bar{U}$ is ¢ as $x \uparrow$

but $U_{max} \uparrow$ as $x \uparrow$

Also eventually δ_m grows from 0 at entrance to $\delta_m = R$ at fully dev. conditions (not considered entrance region any more.

Similar results for thermal $\delta_T(x)$

once we hit the end of the entrance region (we are in the fully developed region) we have

$$\frac{\partial u(r,x)}{\partial x} = 0 \quad \Rightarrow \quad u = u(r) \text{ only}$$

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s - \bar{T}}\right] = 0 \quad \Rightarrow \quad T = T(r) \text{ only}$$

Conseq. of $T = T(r)$ only:

AND $\left.\frac{\partial T}{\partial r}\right|_{r=R} \neq f(x)$

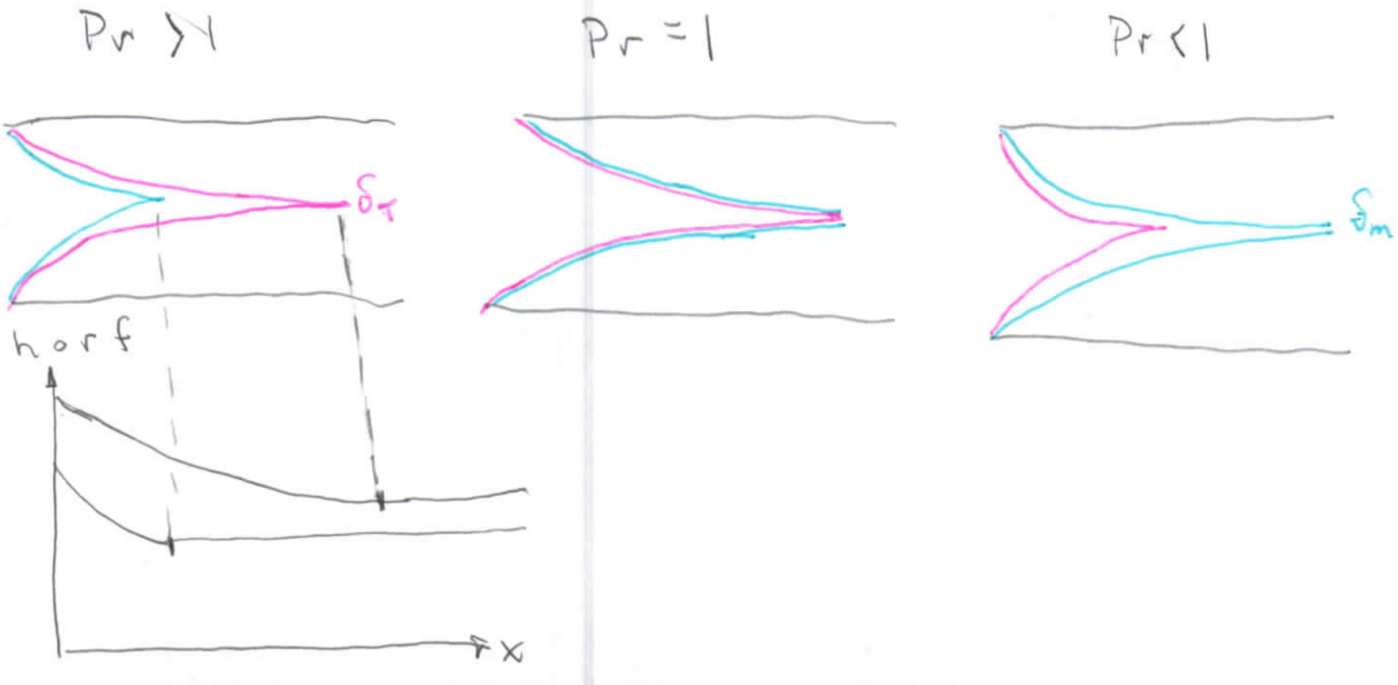
so $\underset{\text{(local value)}}{h(x)} \equiv \frac{k\left(\partial T/\partial r\right)\big|_{r=R}}{T_s - \bar{T}} \neq f(x)$

Thus in the fully developed section

fric factor $f \neq$ function (x)

$h \neq$ function (x)

Role of $Pr = \nu/\alpha$



So what are entry length results?

Laminar flow

$$L_{hyd.} \approx 0.05\ Re\ D$$

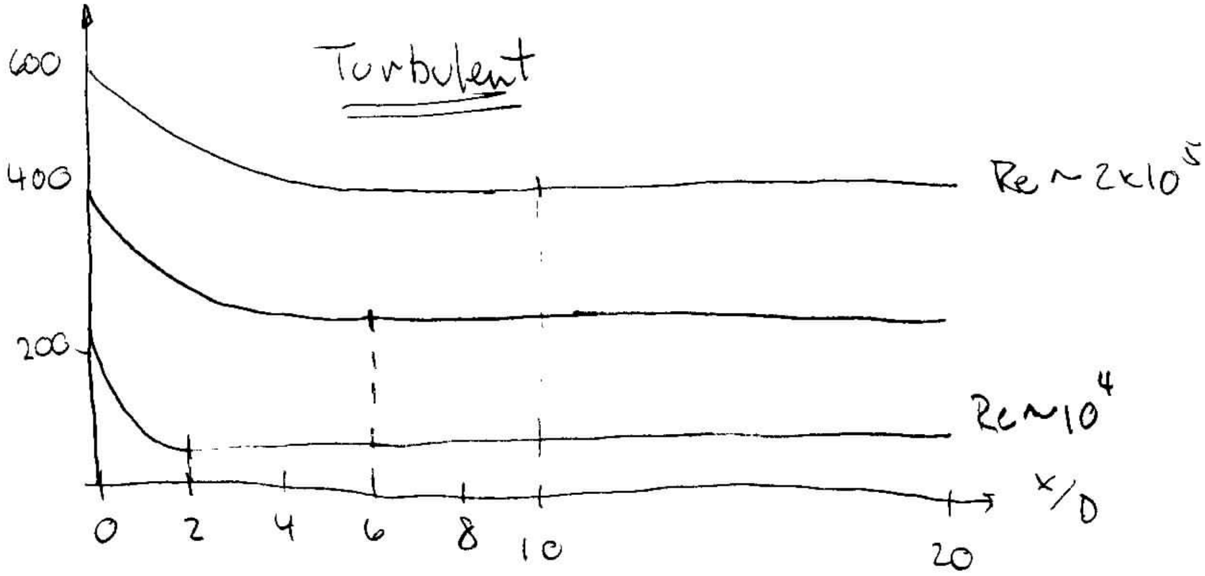
$$L_{therm.} \approx 0.05\ Re\ D\ Pr$$

- Thus if $Re = 20$ the entry length $\approx$ diameter of tube
  if $Re \approx 2300$ " $\approx 111\ D$

Turb. flow

$$L_h \approx L_t \approx 10\ D$$

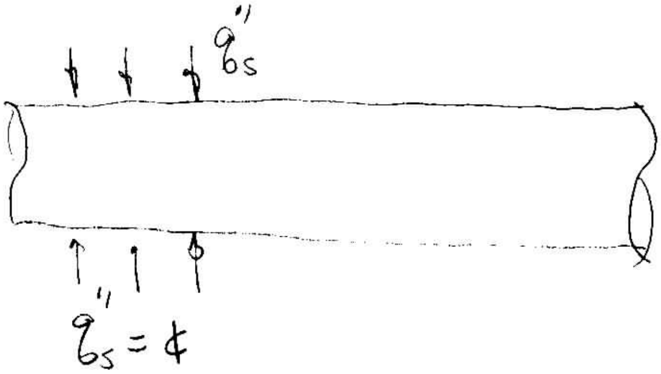
Nu(x) for either $T_s = ¢$ or $q = ¢$ at surface



Observations:
- Nu ↑ in entrance region
- Nu are all $\approx ¢$ for $\frac{x}{D} > 10$ (no matter the Re)
- Nu is almost indep. of B.C. ($T_s = ¢$ or $q_s = ¢$) (ent. or fully dev.)

  ⇒ Might as well just use turb. correlations for either type of B.C.

Constant Heat Flux at surface



Surf. heat flux $\quad q_s'' = h(x)\left(T_s(x) - \overline{T}(x)\right) \quad \left[W/m^2\right]$

q_s'' = ¢ by B.C.

$h(x)$: local heat transf. coef.

$\overline{T}$ changes down the pipe

There can still be lots of things that vary with x on RHS.

$$q = q_s'' A_s = \dot{m} C_p \left(T_{exit} - T_{inlet}\right) \quad [W]$$

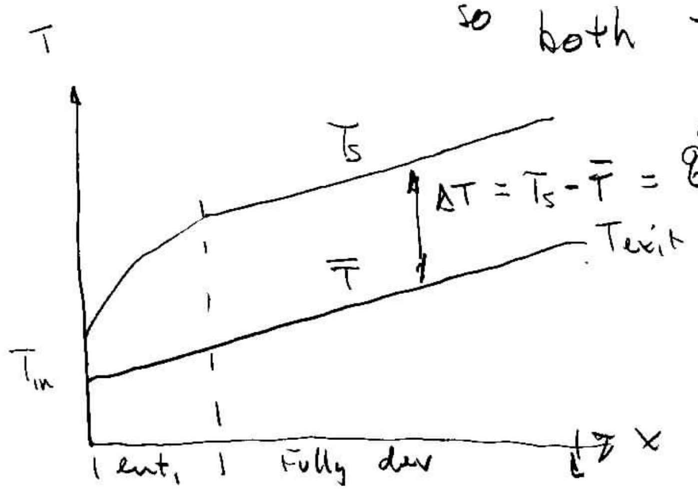
Then $T_{exit} = T_{in} + \dfrac{q_s'' A_s}{\dot{m} C_p}$ $\Rightarrow \overline{T}$ increases linearily in dir. of flow

recall A_s = surf. area
= perim $\cdot$ length

But also $q_s'' = h\left(T_s(x) - \overline{T}(x)\right) \Rightarrow T_s = \overline{T} + \dfrac{q_{surf}''}{h}$

But in fully dev. region $h = ¢$ (and $q_s'' = ¢$)

so both $T_s(x)$ and $\overline{T}(x)$ increase linearly



In fully developed region, the dimensionless $\theta = \frac{T_s - T}{T_s - \bar{T}}$ is ¢ wrt x

so $\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - \bar{T}}\right) = 0 = \frac{\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x}}{(T_s - \bar{T})}$ leads to $\frac{\partial T}{\partial x} = \frac{\partial T_s}{\partial x} = \frac{dT_s}{dx}$

in which case

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \underbrace{\frac{q''_s \cdot \text{Perim}}{\dot{m}\, C_p}}_{\text{from simple heat bal. at a slice through the pipe}} = ¢ \quad (*)$$

None of these gradients (wrt x) vary down the length of the pipe.

$\int (*)$ from $x = 0$ where $\bar{T} = T_{inlet}$ we get

$$\bar{T} = T_{inlet} + \left(\frac{q''_s\, P}{\dot{m}\, C_p}\right) x$$

Eval. this @ $x = L$ (where $\bar{T} = T_{exit}$) and recall $A_s = P \cdot L$

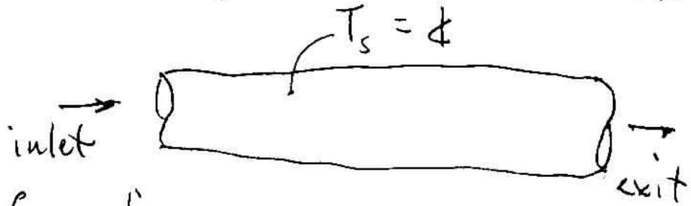
we get $T_{exit} = T_{inlet} + \frac{q''_s A_s}{\dot{m}\, C_p}$

For a circular tube $P = 2\pi R$ $\dot{m} = \rho \bar{U} A_{cross} = \rho \bar{U} (\pi R^2)$

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{d\bar{T}}{dx} = \frac{2 q''_s}{\rho \bar{U} C_p R} = ¢$$

Const q'' flux @ tube wall.

Now what about B.C. of $T_s = ¢$ @ wall?



Roughly speaking, from Newton's law of cooling

$$q = h A_{surf} \Delta T_{avg} = h A_s (T_s - \bar{T})_{\text{average}} \quad [W]$$

h: avg value over surface

$(T_s - \bar{T})$: average spatial ... whatever that is

There are 2 ways of thinking about this,

<u>Arith</u>metic <u>mean</u> <u>temp.</u> <u>differ</u>ence $\Delta T_{a.m.}$

$$\Delta T_{avg} \doteq \Delta T_{a.m.} = \frac{\Delta T_i + \Delta T_e}{2} = \frac{(T_s - T_i) + (T_s - T_e)}{2}$$

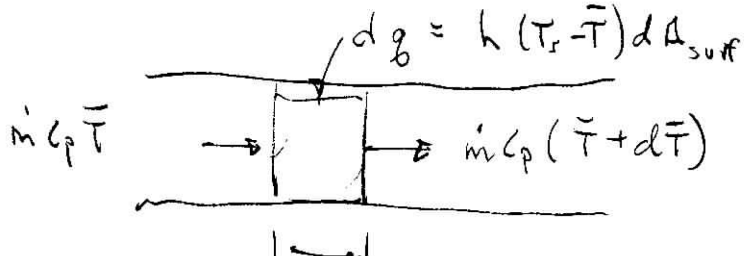
$$= T_s - \left(\frac{T_i + T_e}{2}\right)$$

$$= T_s - T_{bulk}$$

$\frac{\Delta T_i + \Delta T_e}{2}$: arith. average of temp dif. btwn surf and fluid at inlet and exit

leads to reasonable results, but often fails.

Consider an energy balance on



$$\dot{m} c_p \, d\bar{T} = h (T_s - \bar{T})\, dA_{surf}$$

note $dA_{surf} = p \cdot dx$ (p: perim.) and $d\bar{T} = -d(T_s - \bar{T})$

leads to $\dfrac{d(T_s - \bar{T})}{(T_s - \bar{T})} = -\dfrac{hp}{\dot{m}C_p}\,dx$ $\int$ from $x=0$ $(\bar{T} = T_i)$

to $x = L$ $(\bar{T} = T_e)$

leads to

$$\ln\left(\frac{T_s - T_e}{T_s - T_i}\right) = -\frac{hA_s}{\dot{m}C_p} \quad **$$

(h is the ¢ average heat trans. coef.)

exponiate to get

$$\frac{T_e - T_s}{T_i - T_s} = \exp\left(\frac{-hA_s}{\dot{m}C_p}\right)$$ where $A_s = pL$

If you replace $A_s = pL$ by px, you get soln. for

$$\frac{\bar{T}(x) - T_s}{T_i - T_s} = \exp\left(\frac{-hpx}{\dot{m}C_p}\right)$$

We define $\dfrac{hA_s}{\dot{m}C_p} = NTU$ Number of transfer units

If $NTU > 5$ then $T_{exit} \approx T_{surf}$

$NTU > 5$ no more heat trans. happens. No need to add more length.

$NTU < 5$ helps to add more length to pipe

Solve (**) for $\dot{m} c_p = \dfrac{-h A_s}{\ln\left[\dfrac{T_s - T_e}{T_s - T_i}\right]}$

put this into $q = \dot{m} c_p (T_e - T_i)$ [W] near start of today

we get

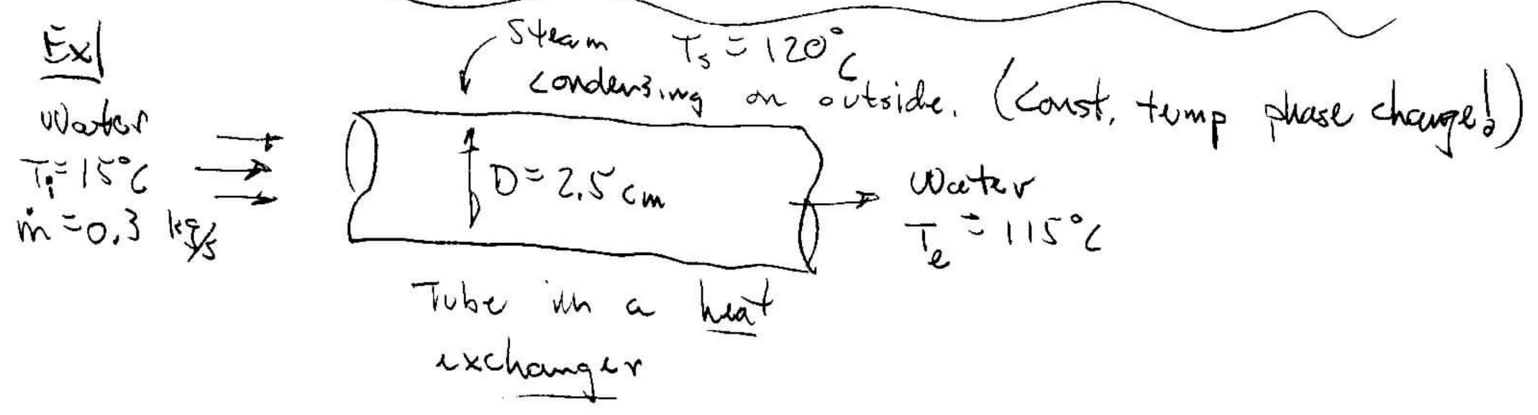
$$q = h A_s \left\{ \frac{(T_i - T_e)}{\ln\left[\dfrac{T_s - T_e}{T_s - T_i}\right]} \right\} = h A_s \Delta T_{lm}$$

where $\Delta T_{\text{log mean}} = \dfrac{T_i - T_e}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} = \dfrac{\Delta T_e - \Delta T_i}{\ln\left(\dfrac{\Delta T_e}{\Delta T_i}\right)}$

<u>Log mean Temp. Dif.</u>

This is the way to go rather than the Arit. mean temp. dif. method.

<u>Ex</u>



Tube in a <u>heat exchanger</u>

If Average $h = 800\ \mathrm{W/m^2 K}$, determine tube length L for exit temp. of $115°C$.

Assumptions:
- Steady state
- h is ¢
- ¢ fluid prop.
- R_{copper} is negl. (inside tube T = outside tube T) k_{cu} ↑↑

Mat'l. prop: @ $T_{bulk} = \frac{(15+115)^\circ C}{2} = 65^\circ C$

$c_p = 4187\ J/kg\cdot K$ phase change heat release @ $120^\circ C$ $= 2203\ \frac{kg}{kg}$

Let's go: for the water in the tube

$q_{trans.} = \dot{m} c_p (T_e - T_i) = \ldots = 125.6\ kW$

To calc ΔT_{lm} we need $\Delta T_e = T_s - T_e = 120^\circ C - 115^\circ C = 5^\circ C$

$\Delta T_i = T_s - T_i = 120^\circ C - 15^\circ C = 105^\circ C$

$$\Delta T_{lm} = \frac{\Delta T_e - \Delta T_i}{\ln\left(\frac{\Delta T_e}{\Delta T_i}\right)} = \ldots = 32.85^\circ C$$

The heat transfer surface are is from

$q = h A_s \Delta T_{lm}$ or $A_s = \frac{q}{h \Delta T_{lm}} = \ldots = 4{,}78\ m^2$

But $A_s = \pi D L$ so $L = \frac{A_s}{\pi D} = 61\ m$

Comments $T_{bulk} = 65^\circ C$ so arithmetic $\Delta T_{am} = (120-65)^\circ C = 55^\circ C$ which would give $L = 36\ m$ which is really wrong!

Use ΔT_{lm} !

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